Flow interpretation implications for Poro-Elastic Modeling

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***Abstract*— The basis of poro-elastic modeling is a set of assumptions about the nature of the porous media and the fluid that fill its void spaces. The basic assumptions are: (1) the solid matrix is homogenous, elastically deformable , and chemically inert with respect to the fluid, (2) the fluid is single phase and Newtonian, (3) the flow is in the laminar range, (4) there is an explicit one-to-one relationship between porosity and permeability, and (5) the fluid flow is governed by Darcy's law. There are two basic interpretations of Darcy's law. The primary interpretation in poro-elastic acoustics is to assume that Darcy's law is a restatement of Poisseuille's law for porous media applications. This restatement can be extended for bundles of capillary tubes, and non-straight tubes. This gives rise to a linear equation that predicts the acoustic dispersion, and attenuation for acoustic waves in a porous media. Unfortunately, simulations of fluid flow in Hele-Shaw cells suggest that for porous media consisting of face-centered-cubically packed spheres the Poisseuille law treatment is incorrect. Both the distribution of fluid velocity, and viscous drag are incorrect.The second interpretation is to assume that Darcy's law is a macro-scale statistically derived flow relationship derived from underlying micro-scale processes. This methods gives rise to a model that inherently includes flow geometry, and variations in permeability and porosity. The statistical model allows for the definition of permeability such that the tortuosity of the fluid paths are explicitly included in both the permeability and the inertial effects. Additional fluid dynamic phenomena, such as inertial effects, internal friction and local accelerations among others can be included in the calculation .**

Two quantities of interest in understanding porous media flow characteristics are the dispersivity of the porous media, and the permeability. The dispersivity determines the spreading of a definite fluid portion, and the permeability determines the average flux of a fluid through the porous media. The Poisseuille law treatment leads to models that can explain longitudinal dispersion in porous media flows, but cannot explain transverse dispersion. The macro-scale statistical model can explain both phenomena at a cost of complexity. Thus the Poisseuille law treatment will give rise to less dispersive estimates of the momentum transfer, and thereby greater overall acoustic effects caused by momentum transfer by the fluid.

The Poisseuille law treatment or the statistical treatment of Darcy's Law can employed within the consolidation model framework to predict compressional phase speeds, shear phase speeds, and attenuations. The predictions from either method are similar. Both methods predict a non-linear dependence on frequency, frequency dependent phase speeds, and two compressional waves. The overall shape of the sound speed dispersion relationship for each of the fluid flow models is similar, although a significant difference is obvious. The most significant difference in the predictions from the two interpretations is the velocity difference between high-frequency and low-frequency phases speeds is larger by roughly a factor of two for the Poisseuille law interpretation. This discrepancy has implications for the interpretation of acoustical inversions based on poro-elastic models.

I. Introduction

Poro-elastic models for acoustic propagation in sediments arose out of the familiar consolidation models for predicting transient fluid flows in soils [1][2]. The Biot-Stoll [3][4] poro-elastic model treats wave propagation in a porous elastic media using macroscopic continuum mechanics. The fluid saturating the porous media is assumed to interact with the porous through a combination of viscous drag and inertia interaction. The viscous interaction is treated by a Darcy's law formulation modified for oscillating flow in an otherwise Poiseuille dynamical range [5]. The assumption of fluid flow in straight constant cross section conduits is the single most critical assumption in defining the relationship between frequency and attenuation. The assumption leads to a frequency dependent permeability that increases as the square root of the frequency [6]. The model assumes that only three parameters are needed to specify the hydrodynamic flow, the porosity η, the permeability k, and an added mass coefficient. The Biot-Stoll model predicts the existence of two compressional waves, and a single shear wave. The assumption of Poiseuille flow in the void structure of the porous media determines the relationship of compressional phase speed to frequency as well as controlling the relationship between attenuation and frequency.

In this paper some of the effects of assuming a non-Poiseuille flow will be elucidated. The Biot-Stoll [3] model is used as the starting point. This forumulation give the essential strucuture of the classic Biot fluid dynamic treatment. The fluid dynamic treatment in the model is modified to include inertial effects with the explicit assumption that the maximum flow length before a branch occurs is on the order of the mean particle size of the matrix. That is, Darcy's law is not a restatement of Poiseuille's law for a porous media.

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14. ABSTRACT
The basis of poro-elastic modeling is a set of assumptions about the nature of the porous media and the fluid that fill its void spaces. The basic assumptions are: (1) the solid matrix is homogenous, elastically deformable , and chemically inert with respect to the fluid, (2) the fluid is single phase and Newtonian, (3) the flow is in the laminar range, (4) there is an explicit one-to-one relationship between porosity and permeability, and (5) the fluid flow is governed by Darcys law. There are two basic interpretations of Darcys law. The primary interpretation in poro-elastic acoustics is to assume that Darcys law is a restatement of Poisseuilles law for porous media applications. This restatement can be extended for bundles of capillary tubes, and non-straight tubes. This gives rise to a linear equation that predicts the acoustic dispersion, and attenuation for acoustic waves in a porous media. Unfortunately, simulations of fluid flow in Hele-Shaw cells suggest that for porous media consisting of face-centered-cubically packed spheres the Poisseuille law treatment is incorrect. Both the distribution of fluid velocity, and viscous drag are incorrect.The second interpretation is to assume that Darcys law is a macro-scale statistically derived flow relationship derived from underlying micro-scale processes. This methods gives rise to a model that inherently includes flow geometry, and variations in permeability and porosity. The statistical model allows for the definition of permeability such that the tortuosity of the fluid paths are explicitly included in both the permeability and the inertial effects. Additional fluid dynamic phenomena, such as inertial effects, internal friction and local accelerations among others can be included in the calculation .

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II. EXTENDED POISEUILLE TREATMENT

The theoretic hydrodynamic aspects of a single homogenous fluid in a isotropic nonreactive saturated medium form a critical foundation for the Biot model. When the pore space is in the form of a network of tubes, with distances between the nodes greater than the transverse dimensions of the tubes, it seems reasonable to treat the system as an equivalent Poiseuille tube network. The theoretical approach of replacing the porous media by an 'equivalent' bundle of capillary tubes was taken by Biot as the basis of his fluid dynamic analysis [5]. That is, the Darcy Law relationship

$$\nabla P = -\frac{\mu}{k} u \qquad (1)$$

where μ is the dynamic viscosity of the fluid, k is the permeability, u is the macroscopic velocity and P is the pressure, is assumed to refer to an exactly linear flow in a precise geometry in the same manner as the Poiseuille Law. The momentum balance is assumed to be consistent with steady unidirectional flow with the pressure gradient parallel to the flow direction, and thus, for a porous media the conduits are parallel to the pressure gradient. The assumption of Darcy's Law as a statement of Poiseuille flow for a porous media allows the development of frequency dependent viscosity [5]. The extension assumes that the velocity vector has the same direction everywhere, and is independent of distance in the flow direction,thus the material derivative in the momentum equation is identically zero. This is achieved by assuming the flow occurs in constant geometry conduits. Solutions of this equation, assuming that Darcy's Law is the relationship between pressure and velocity and pressure gradient, give rise to a term that is often interpretable as frequency dependent viscosity. The dynamic viscosity η of the fluid is replaced by $\eta F(\kappa)$, where $F(\kappa)$ describes the deviation from Poiseuille flow friction as a function of a frequency parameter κ. For low frequencies the limit is 1, and for high frequencies it is proportional to the square root of the frequency and is 45 degrees out of phase with respect to the velocity. The resultant model predicts the existence of three forms of body waves, two compressional waves, and one shear wave. The existence of the slow wave is not dependent on the details of the porous media treatment, but the attenuation is dependent on the treatment as well as the velocity difference between the low frequency limit and the high frequency limit for all wave types. The critical assumption is that flow in a porous media occurs through straight, smooth conduits. Analog evidence [7] suggests that deviations from the smooth, straight conduit when the pressure gradient is not parallel to the flow path leads rapidly to lower flow efficiencies, and non-Poiseuille flow.

III. STATISTICAL DARCY'S LAW

Rather than assume that flow in a porous media occurs along well defined paths that are parallel to the pressure gradient, assume a model where the momentum equation is a statement of the average state of a representative volume. In simple porous media the individual pores can be treated as volumes connected between constrictions in the flow path. The distance between constrictions is commonly on the order as the transverse dimensions of the pore [8]. The fluid flow in such a media is then characterized by divergences and convergences with length scales on the order of an individual pore. For this porous media model, [9] used a Stokes flow solution to demonstrate the difference between a Poiseuille flow and the flow encountered in a porous media. Three significant differences were discovered, (a) the maximum vorticity and shear stress is not always found at the pore wall (Poiseuille flow has maxima of both quantities at the the wall),(b) there is no one to one relationships between point values of potential gradient and flow velocity as required by Poiseuille flow, and (c) the velocity deviates strongly from the Poiseuille parabolic profile. Such flow behavior suggests that the Poiseuille flow assumption made by [10] is invalid, unless the porous media has a specific capillary flow geometry.

For a non-Poiseuille flow, the [10] extension to oscillating flows is unavailable. However, the flow is being forced by an acoustic oscillation that requires time-dependency in the flow equations. For mean porous media flow in which the only significant inertial term is the local time dependency of the flow we use a formulation found in [11]. That is

$$\nabla P = -\frac{\mu}{k} u - \frac{\rho_f}{T}\frac{\partial u}{\partial t} \qquad (2)$$

where ρ_f is the fluid density, and T is the tortuosity of the porous media. This equation is recognizable as Darcy's law with the addition of term relating local acceleration to the momentum balance. The form of this equation is similar to the equation solved by Biot [5], the difference between the two equations lies : 1) in the constants, and 2) in the assumption of the flow paths. In this formulation permeability is proportional to tortuosity ([8], [11]). The permeability is the product of porosity, terms relating to the physical structure of the porous media, and the tortuosity. The second term in the equation becomes significant when the Strouhal number (the ratio of vibration speed to flow velocity) becomes greater than 1. Since acoustic pressures are small (relative to the overburden pressure) the velocities excited by the acoustic waves are small, thus the vibrational speed (frequency times characteristic length) is much larger than the fluid velocity. Tortuosity is defined as

$$T = (\frac{l}{l_e})^2 \qquad (3)$$

where l is the distance between two points, and l_e is the length of the flow path connecting the points. This specification is limited to the case of laminar flow of a Newtonian fluid through a porous media. The media consists of a network of channels in which the flow occurs. The channels are then stream tubes fixed in space, that is streamlines in the porous media have a fixed geometry. In this formulation a straight flow tube would have a tortuosity of 1, with more convoluted paths having a smaller value. The permeability is maximized when the flow tubes are straight, and decreases as the flow path increases. Values for tortuosities [8] for various geometries suggest a value around 0.4 would be a lower limit.

IV. Comparitive Calculation

The methods outlined in sections 2 and 3 will now be used to calculate the phase speed, and qualtity factor as a function of frequency. The example is taken from , and the inputs are given in table 1. The Biot-Stoll model uses an extra mass of 0.25 in the calculation. In the advective diffusive approach the effective method of creating extra mass is to set the tortuosity such that the factor 1/T. The method gives rise to a term that yeilds the extra mass needed.

Each of the flow models was used to calculate the poro-elastic results for frequencies from 10 Hz to 240 kHz. The results are presented in figures 1 through 3. Each figure consists of the phase speed of the wave type, and the specificate attenuation of the wave type for each of the two fluid flow models.

Figure 1 shows the phase speeds, and specificate attenuations for the Biot-Stoll flow model, and the advective-diffusive flow model for the fast compressional wave. Each of the predictions show a number of common features. First the phase speed is minimal at low frequencies rising quickly in the low kHz range, and reaching a maximal in the low hundreds of kHz. Second the shape of the specificate attenuation curves is similar for both flow models. The differences can be characterized into five categories: (1) the difference between the low frequency phase speed and high frequency phase speed is twice as large for the Biot-Stoll flow model, (2) the range over which phase speed increases rapidly with frequency is larger in the Biot-Stoll flow model, (3) the specificate attenuation is higher in the Biot-Stoll model at all frequencies, (4) the rate of decay in the specificate attenuation after the peak specificate attenuation is much smaller when using the Biot-Stoll model, and (5) the maximum slope of either the phase speed dispersion curve or specific attenuation is larger for the Poiseuille based flow.

Figure 2 shows the phase speeds, and specificate attenuations for the Biot-Stoll flow model, and the advective-diffusive flow model for the slow compressional wave. The points of commonality, and difference are similar for the slow and fast compressive waves. The difference between the low and high frequency phase speeds is roughly a factor of two , with the greater difference arising for the Biot-Stoll flow regime. The frequency interval over which the phase speed rises rapidly with frequency is larger for the Biot-Stoll flow regime. The quailty factors are almost equal to frequencies near 2 kHz, but for higher frequencies the specificate attenuation of the Biot-Stoll flow regime decays less quickly than the advective-diffusive flow regime.

Figure 3 shows the phase speeds, and specificate attenuations for the Biot-Stoll flow model, and the advective-diffusive flow model for the shear wave. The shear wave differences are very similar to the differences observed in the compressional waves with one significant difference. The magnitude of the differences are reduced due too the relative dominance of the elastic shear terms to the poro-elastic shear terms. The specific attenuation, and phase speed of the Poiseuille based model is higher than the advective-diffusive based model except for a small interval between 3 and 10 kHz when the phase speed, and the specific attenuation are higher for the advective-diffusive base model. This is the also the only region in which the gradient of these properties is higher for the advective-diffusive based model.

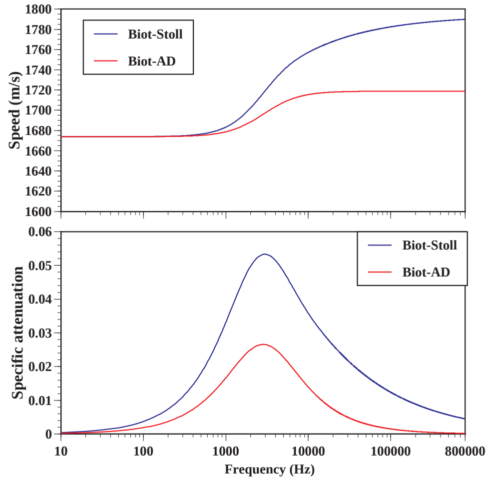

Fig. 1. Plot of wave speed versus frequency , and specific attenuation versus frequency for fast compresssional wave using Biot-Stoll and Biot-advective diffusion poro-elastic model.

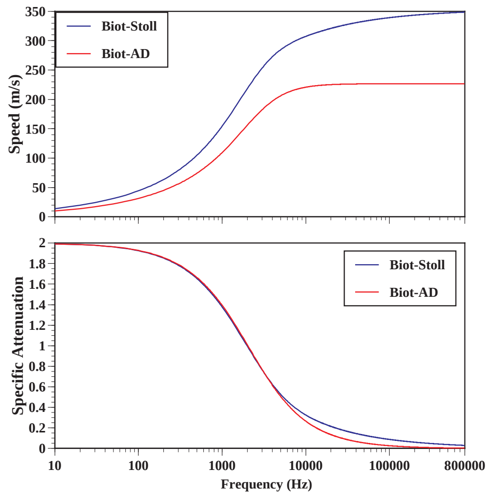

Fig. 2. Plot of wave speed versus frequency , and specific attenuation versus frequency for slow compresssional wave using Biot-Stoll and Biot-advective diffusion poro-elastic model.

V. Discussion and Conclusions

The comparison of the poro-elastic model predictions using the Poiseuille and advective diffusive models reveal :

1) Both flow regimes allow for the existence of slow and fast compressional waves. This is implicit in the formulation of the Biot consolidation model [2] which stipulates that the fluid flow is defined relatively to the matrix.

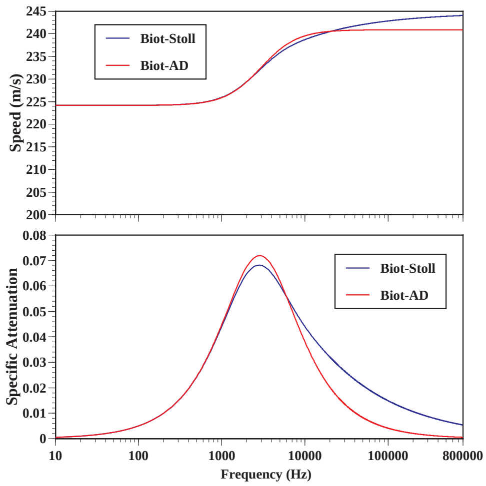

Fig. 3. Plot of wave speed versus frequency , and specific attenuation versus frequency for shear wave using Biot-Stoll and Biot-advective diffusion poro-elastic model.

TABLE I
PHYSICAL PROPERTIES OF SEDIMENTS.

Physical Property	Symbols	units	Value
Fluid Bulk Modulus	k_f	(Nm^{-2})	$2.3 \cdot 10^9$
Frame Bulk Modulus	k_b	(Nm^{-2})	$2.67 \cdot 10^8$
Grain Bulk Modulus	k_r	(Nm^{-2})	$3.6 \cdot 10^{10}$
Frame Shear Modulus	μ_b	(Nm^{-2})	$1.0 \cdot 10^8$
Porosity	η		0.40
Viscosity	μ	$kgm^{-1}sec^{-1}$	$1.0 \cdot 10^3$
Permeability	k	(m^2)	$1.0 \cdot 10^{-10}$
Fluid density	ρ_f	(kg/m^3)	$1.0 \cdot 10^3$
Density	ρ	(kg/m^3)	$1.99 \cdot 10^3$

2) The same basic shape of phase velocity dispersion curve, and specific attenuation dispersion curve is predicted by both flow models.

3) The difference between the high frequency phase speed and the low frequency phase speed is approximately twice as large for the Poiseuille based flow model poro-elastic predictions.

4) The specific attenuation is generally higher for the Poiseuille base flow model poro-elastic predictions.

5.) The region where the phase speeds exhibit the greatest frequency dependency is larger for the Poiseuille based flow model poro-elastic predictions.

6.) The slopes of the phase speed dispersion are larger for Poiseuille based poro-elatic models.

The physical mechanism for the differences arises for the dispersion characteristics of the assumed fluid flow. Poiseuille flow models in porous media gives rise to models which cannot account for transverse dispersion of mass, energy and momentum[8][11][9]. This means that the acoustic energy is more coupled with the fluid motion in the matrix than would be expected. This means for a complex valued wave for energy would be moved into both elements of the complex wave giving rise to the paradox of higher specific attenuation.

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